3. M. Hoffmeister, "Using a single hot-wire probe in three-dimensional turbulent-flow fields," DISA Inf., No. 13, 26-29 (1972).
4. A. L. Genkin, V. I. Kukes, and L. P. Yarin, "Toward a method of measuring turbulent velocity pulsations in nonisothermal flows," Inzh.-Fiz. Zh., 35, No. 4, 651-654 (1978).
5. V. V. Fedorov, Optimum Experiment Theory [in Russian], Nauka, Moscow (1971).
6. M. Acrivlellis, "Some remarks on methods of evaluating hot-wire output using squared signals," DISA Inf., No. 24, 40-42 (1979).
7. V. I. Kukes, A. A. Ufimtsev, and L. P. Yarin, "Method of thermoanemometric measurements in high-intensity turbulence," Second All-Union Conference on Methods of Aerophysical Investigations (collection of documents), Institute of Theoretical and Applied Mechanics, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk, Vol. 2 (1979), pp. 17-19.

CLOSED EQUATION FOR THE STRUCTURE FUNCTION OF AN ISOTROPIC TURBULENT VELOCITY FIELD

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A closed equation is derived for the structure function of an isotropic turbulent velocity field in an incompressible fluid. The equation for the characteristic function [1] is used as the initial equation.

A closed equation was obtained in [1] for the characteristic function φ of the probability distribution of the differences in velocities and temperatures at two points in an isotropic turbulent flow of an incompressible fluid. Here, we use this equation as the initial equation to derive an equation for the structure function, which is defined as follows:

$$D(r,\ t) = \langle \Delta V_l(r,\ t)^2 \rangle . \tag{1}$$

The time-dependent argument of the structure functions will not be indicated in subsequent discussions.

Using the equation for φ, we can also obtain an equation for the structure function of the temperature field H(r, t). This equation will be derived in the present article. In making the transition from the equation for φ to the equations for D and H, there again arises the problem of closure. To obtain closed equations for D and H, we need to make certain assumptions regarding the form of the characteristic function φ. We will choose for the form of φ the product of the Gaussian characteristic function and an expression accounting for the deviation of the combined probability distributions of the velocity and temperature differences from the normal distribution. This expression will contain only those moments of the probability distribution which have an important physical significance: 1) the double-point third-order structural tensor $D_{ijl}(r)$ describing the transfer of energy between different-scale pulsations of the turbulent velocity field,

$$D_{ijl}(\mathbf{r}) = \langle \Delta V_i(\mathbf{r}) \Delta V_j(\mathbf{r}) \Delta V_l(\mathbf{r}) \rangle ; \tag{2}$$

2) a mixed third-order moment defining turbulent mixing of the temperature field

$$D_{jTT}(\mathbf{r}) = \langle \Delta V_j(\mathbf{r}) \Delta T^2(\mathbf{r}) \rangle . \tag{3}$$

Thus, the assumption made with regard to the form of φ consists of the following:

$$\varphi_{\mathbf{r},t}(\theta,\ \eta) = \exp \Gamma(\theta,\ \eta) [1 - iT(\theta,\ \eta)], \tag{4}$$

where

$$\Gamma(\theta,\ \eta) = -\frac{1}{2} D_{ij}(\mathbf{r}) \theta_i \theta_j - \frac{1}{2} H(\mathbf{r}) \eta^2; \tag{5}$$

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$$T(\theta, \eta) = \frac{1}{6} D_{ijl}(\mathbf{r}) \theta_i \theta_j \theta_l + \frac{1}{2} D_{jTT}(\mathbf{r}) \theta_j \eta^2. \quad (6)$$

For the case of an isotropic solenoidal field, the tensor $D_{ij}(r)$ can be expressed through a single scalar function. Determined by means of a longitudinal structural function (Dr) (1), it has the form [2]

$$D_{ij}(\mathbf{r}) = -\frac{r}{2} D'(r) \nu_i \nu_j + \left[D(r) + \frac{r}{2} D'(r) \right] \delta_{ij}. \quad (7)$$

The tensor $D_{ijl}(r)$ for the case of an isotropic solenoidal field is expressed as follows [2]:

$$D_{ijl}(\mathbf{r}) = \frac{1}{2} [M(r) - rM'(r)] \nu_i \nu_j \nu_l + \frac{1}{6} [M(r) + rM'(r)] (\nu_i \delta_{jl})_s, \quad (8)$$

where

$$M(r) = \langle \Delta V_L(r)^3 \rangle . \quad (9)$$

The subscript s denotes that the expression in brackets should be balanced with respect to all indices (i, j, l).

The form adopted for the characteristic function (4)-(6) is equivalent to assuming that the turbulent velocity and temperature fields have a quasinormal structure. Such an approximation has been used repeatedly in elaborating the theory of turbulence based on moment formalism [2]. Several experiments [3-5] have been devoted to this hypothesis. The failures encountered in this approach (particularly the negative sections of the spectrum at large Reynolds numbers) are evidently attributable not to the fact that the actual probability distributions are far from quasinormal but, more likely, to the fact that the quasinormality hypothesis has not been correctly incorporated into formal theory. When the hypothesis is properly applied, these problems do not arise [6].

Let us substitute the expression for φ in the form (4)-(6) into the right side of Eq. (33) in [1] and obtain the following expression:

$$\varphi_{\mathbf{r},t}(\theta, \eta) = \frac{1}{(2\pi)^3} \int_0^t \frac{d\tau}{(t-\tau)^3} \int d\mathbf{y} \exp\left[i\theta \cdot \frac{(\mathbf{r}-\mathbf{y})}{t-\tau} - \frac{1}{2} H(y) \eta^2 \right] \times$$
$$\times \int d\theta' \cdot \exp\left[-i\theta' \cdot \frac{(\mathbf{r}-\mathbf{y})}{t-\tau} - \frac{1}{2} D_{ij}(\mathbf{y}) \theta'_i \theta'_j \right] L(\theta', \eta, \mathbf{y}, \tau)[1 - iT(\theta', \eta)]. \quad (10)$$

A formula connecting D(r) with $\varphi_{\mathbf{r},t}(\theta, \eta)$ can be obtained by using the definition (1) from [1] and Eq. (7):

$$D(r) = \hat{D} \varphi_{\mathbf{r},t}(\theta, \eta). \quad (11)$$

The operator $\hat{D}$ acts as follows:

$$\hat{D} f \equiv -[(\nu \cdot \nabla_\theta)^2 f]_0. \quad (11')$$

Applying the operator $\hat{D}$ to the left and right sides of (10), we obtain an equation for D(r):

$$D(r) = \frac{1}{(2\pi)^3} \int_0^t \frac{d\tau}{(t-\tau)^5} \int dy [\nu(\mathbf{r}-\mathbf{y})]^2 \int d\theta \exp\left[-i\theta \frac{(\mathbf{r}-\mathbf{y})}{t-\tau} - \frac{1}{2} D_{ij}(\mathbf{y}) \theta_i \theta_j \right] \hat{\Omega}(\theta), \quad (12)$$

where

$$\hat{\Omega}(\theta) = \Omega_{\alpha\beta}(\mathbf{y}) \theta_\alpha \theta_\beta + i\Omega_{\alpha\beta\gamma}(\mathbf{y}) \theta_\alpha \theta_\beta \theta_\gamma + \frac{1}{6} D_{\delta kl}(\mathbf{y}) \Omega_{\alpha\beta\gamma}(\mathbf{y}) \theta_\delta \theta_k \theta_l \theta_\alpha \theta_\beta \theta_\gamma. \quad (13)$$

In the writing of (12), only terms which are proportional to the first powers of the quantities ε_d, ν, and $(r/L)^2$ remain.

Equation (12) is not closed, since it contains the tensor $D_{\delta kl}(r)$ in the right side, i.e., a third-order moment M(r). If we use Eq. (8) from [1] for the characteristic function φ, we can obtain a relation connecting the functions M and D. Acting on the left and right sides of (8) from [1] with the operator $\hat{D}$, we find

$$D'_t(r) + \nabla_{r_l} D_{ijl}(\mathbf{r}) \nu_i \nu_j = -2\Omega_{ij}(\mathbf{r}) \nu_i \nu_j. \quad (14)$$

Using Eqs. (12), (15), and (18) from [1] — which expand the sense of the tensor Q_{ij} — and (8), this relation may be reduced to the following form:

$$M(r) = -\frac{4}{5}\varepsilon_d r + \frac{2}{7}\left(\frac{r}{L}\right)^2 \varepsilon r + 6\nu D'(r) - \frac{3}{r^4}\int_0^r x^4 D_t'(x)\,dx. \tag{15}$$

This is the exact result of the equations of motion for the case of isotropic turbulence at $r \ll L$, although we obtained it from an approximate closed equation for φ. Equation (15) was obtained without the last term in [7] and without the term $\infty (r/L)^2$ in [8].

Let us substitute the function M in the form (15) into the right side of (8) for $D_{\delta kl}(y)$ and find an expression for this tensor through the structure function D(y) and the parameters of the problem:

$$D_{\delta kl}(\mathbf{y}) = -\frac{4}{15}\varepsilon_d r \rho t_{\delta kl} - \frac{2}{7}\varepsilon r\left(\frac{r}{L}\right)^2 \rho^3\left[p_{\delta kl} - \frac{2}{3}t_{\delta kl}\right] - 3\frac{\nu}{r}\rho D''(\rho r)\left[\tau_1 p_{\delta kl} + \tau_2 t_{\delta kl}\right] + \rho r D_\tau'(\rho r)\left[s_1 p_{\delta kl} + s_2 t_{\delta kl}\right]. \tag{16}$$

Here

$$p_{\delta kl} = \lambda_k \lambda_\delta \lambda_l; \quad t_{\delta kl} = (\lambda_\delta \delta_{kl})_s; \tag{17}$$

$$\tau_1 = 1 + \beta^{-1}(\rho,\ r); \quad \tau_2 = \frac{1}{3}\left[-1 + \beta^{-1}(\rho r)\right]; \tag{18}$$

$$s_1 = \frac{3}{2} - 5A; \; s_2 = A; \quad A = \frac{3\int_0^{\rho r} x^4 D_\tau'(x)\,dx}{2(\rho r)^5 D_\tau'(\rho r)}. \tag{19}$$

Equation (13), with the expression for the tensor $D_{\delta kl}(y)$ in the form (16)-(19), is closed relative to the structure function D(r). All that needs yet to be done is calculation of the integrals with respect to the same variables on which the integrand in the right side of (12) explicitly depends.

To integrate with respect to the variable Θ, Eq. (13) is conveniently written in the form

$$D(r) = \int_0^t \frac{d\tau}{(t-\tau)^5}\int d\mathbf{y}\,[\mathbf{v}(\mathbf{r}-\mathbf{y})]^2 F_{\frac{r-y}{t-\tau}}\left\{\hat{\Omega}(\theta)\exp\left[-\frac{1}{2}D_{ij}(\mathbf{y})\,\theta_i\theta_j\right]\right\}, \tag{20}$$

where $F_{\mathbf{B}}\{A\} = \frac{1}{(2\pi)^3}\int d\boldsymbol{\theta}\exp[-i\mathbf{B}\boldsymbol{\theta}]A$; $\hat{\Omega}(\theta)$ is a polynomial in powers of $\boldsymbol{\theta}$. Thus, calculation of the integral with respect to the variable $\boldsymbol{\theta}$ reduces to calculation of the Fourier transform from the product of an exponential function and a polynomial. The result of this operation may be represented in the form

$$D(r) = \frac{r^3}{(2\pi)^{3/2}}\int_0^t \frac{d\tau}{(t-\tau)^3}\int_0^{2\pi} d\varphi\int_0^\infty d\rho\rho^2\int_{-1}^1 ds\left\{f_1(\rho/s) + \frac{r^3}{\rho}\int_0^{2\pi} d\psi\int_0^\infty d\sigma\int_{|\rho-\sigma|}^{\rho+\sigma} d\xi\,\xi f_2(\rho,\ \sigma,\ \xi/s,\ l,\ t,\ \delta,\ \gamma,\ \omega)\right. \tag{21}$$

The outer integrals in (21) constitute the expanded three-dimensional integral with respect to y in Eq. (13), while the inner integrals make up the expanded three-dimensional integral $\iint$dxdy, which is contained in the definition (13) of the tensor $\Omega_{\alpha\beta\nu}(y)$ in [1]. It is transformed to this form by the method applied in the work [9]. The expressions f_1 and f_2 in (21) depend on two groups of arguments. Expressions f_1 and f_2 depend on the first group of arguments explicitly and through the sought function D(r). They depend only explicitly on the second group. The variables δ, γ, and ω are the cosines of the angles of a triangle formed by the vectors y, x, and z. They can be expressed either through the lengths of the sides of this triangle or through the relative lengths ρ, σ, and ξ. The variables t, s, and l are the cosines of the angles between the vector r and the vectors y, x, and z, respectively. The variables t and l can be expressed through other variables with the formulas

$$t = s\delta - \sqrt{1-s^2}\sqrt{1-\delta^2}\cos\psi, \quad l = s\gamma - \sqrt{1-s^2}\sqrt{1-\gamma^2}\cos\psi.$$

After this, we can integrate with respect to the variables ψ and s. It is simple to integrate with respect to the variable φ since the integrand is independent of φ. For the part of the expression f_2 connected with $c\omega_{\alpha\beta}^{(1)}$ in the definition (13) of the tensor $\Omega_{\alpha\beta\gamma}(y)$ in [1], it turns out that it is possible to calculate the integral with respect to ξ.

The above transformations, fairly cumbersome but simple in principle, lead to the following equation for D( ):

$$D(r)=r^2\sqrt{\frac{2}{\pi}}\int_0^t\frac{d\tau}{(t-\tau)^3}\int_0^\infty d\rho\rho^2\frac{1}{D^{3/2}(\rho r)}\hat{D}(\rho,\ r,\ \tau), \tag{22}$$

where

$$\hat{D}(\rho,\ r,\ \tau)=\sum_{l=1}^{2}k_0^l(\rho,\ n,\ q)\left[\frac{\nu}{r}D_\rho''(\rho r)\eta_{l1}+\varepsilon r\left(\frac{r}{L}\right)^2\rho^2\eta_{l2}+\right.$$
$$\left.+\frac{\nu}{r}D_\rho''(0)\eta_{l3}\right]-\frac{1}{16D^{1/2}(\rho r)}\sum_{j=1}^{3}\left\{k_j(\rho,\ n,\ q)-\frac{1}{6D^{3/2}(\rho r)}\times\right.$$
$$\left.\times\sum_{l=1}^{2}k_j^l(\rho,\ n,\ q)\left[\frac{\nu}{r}D_\rho''(\rho r)\tau_{l1}+\varepsilon r\left(\frac{r}{L}\right)^2\rho^2\tau_{l2}+\frac{\nu}{r}D_\rho''(0)\tau_{l3}-rD_\tau'(\rho r)\tau_{l4}\right]\right\}M_j(\rho r). \tag{23}$$

Here

$$M_j(\rho r)=\int_0^\infty d\sigma\sigma D_\sigma'(\sigma r)\left\{\left[a^j\left(\frac{\sigma}{\rho}\right)D_\sigma'(\sigma r)-b^j\left(\frac{\sigma}{\rho}\right)\sigma D_\sigma''(\sigma r)\right]+\right.$$
$$\left.+\frac{1}{32\rho^2}\sum_{n=1}^{4}\sum_{k=-6}^{7}\gamma_k^{jn}\left(\frac{\sigma}{\rho}\right)\int_{|\rho-\sigma|}^{\rho+\sigma}d\xi\xi D_\xi'(\xi r)\beta_n(\sigma r,\ \xi r)\left(\frac{\xi}{\rho}\right)^k\right\}. \tag{24}$$

We used the following notation in this equation

$$\eta_{lp}=\begin{vmatrix}2\eta_1 & \frac{1}{3} & 0\\ 2\eta_2 & -\frac{1}{3} & 5\end{vmatrix};\quad \tau_{lr}=\begin{vmatrix}3\tau_1 & \frac{2}{7} & 0 & s_1\\ 3\tau_2 & -\frac{4}{21} & 2 & s_2\end{vmatrix}; \tag{25}$$

$$l=1,\ 2;\ p=1,\ 2,\ 3;\ r=1,\ \dots,\ 4;$$

$$q=r/\sqrt{2}\cdot n\cdot D^{1/2}(y)(t-\tau); \tag{26}$$

$$n^2=1+\{y^2[\ln D(y)]_y'\}^{-1}. \tag{27}$$

The functions η_1 and η_2 are given by Eqs. (16) and (17) in [1], τ_1 and τ_2 are given by Eq. (18), and s_1 and s_2 — by Eq. (19). The expression for the functions k_j^l, j = 0, 1, 2, 3; l = 1, 2, has the form

$$k_j^l(\rho,\ n,\ q)=\frac{n^2-1}{n^2}\sum_{k=0}^{5}\sum_{m=1}^{4}\{a_{jk}^{lm}\psi(\rho,\ n,\ q)-\exp[-q^2n^2(1+\rho^2)]\times$$
$$\times[b_{jk}^{lm}\operatorname{ch}(2q^2n^2\rho)+c_{jk}^{lm}\rho\operatorname{sh}(2q^2n^2\rho)]\}q^{2m-2}\rho^{2k}, \tag{28}$$

where

$$\psi(\rho,\ n,\ q)=\frac{\sqrt{\pi}}{2q}\exp[q^2(n^2-1)(\rho^2n^2-1)]\{\Phi[q(1-\rho n^2)]+\Phi[q(1+\rho n^2)]\}. \tag{29}$$

Here $\Phi(x)$ is the probability integral; sh x, ch x, hyperbolic sine and cosine.

The elements of the matrices a_{ik}^{lm}, b_{jk}^{lm}, c_{jk}^{lm} are presented in Appendix A.

The expression for $k_j(\rho;\ n;\ q)$ has the form

$$k_j(\rho,\ n,\ q) = \sqrt{2}\left(n - \frac{1}{n}\right)\sum_{k=0}^{3}\sum_{m=1}^{3}\left\{a_{jk}^{m}\psi(\rho,\ n) + \frac{1}{q^2}\exp\left[-q^2n^2(1+\rho^2)\right]\times\right.$$

$$\left.\times\left[b_{jk}^{m}\frac{1}{\rho}\,\mathrm{sh}\,(2q^2n^2\rho) + c_{jk}^{m}\,\mathrm{ch}(2q^2n^2\rho)\right]\right\}q^{2m-1}\rho^{2k}. \qquad (30)$$

The elements of the matrices a_{jk}^{m}, b_{jk}^{m}, c_{jk}^{m} are presented in Appendix A. The matrices $a^j(x)$ and $b^j(x)$ are determined as follows:

$$(a^j,\ b^j) = \begin{cases}(a^j_{\langle},\ b^j_{\langle}), & \text{if} \quad x < 1,\\ (a^j_{\rangle},\ b^j_{\rangle}), & \text{if} \quad x > 1,\end{cases} \qquad (31)$$

$$a^j_{\langle} = 2x\begin{Vmatrix} 2 - \frac{12}{5}x^2 \\ -10 \\ -2 + \frac{24}{5}x^2 \end{Vmatrix},\quad a^j_{\rangle} = -\frac{32}{5}\frac{1}{x^2}\begin{Vmatrix} 2 \\ 5 \\ 1 \end{Vmatrix},$$

$$b^j_{\langle} = 4x\begin{Vmatrix} -\frac{2}{3} + \frac{1}{5}x^2 \\ \frac{10}{3} \\ \frac{2}{3} - \frac{2}{5}x^2 \end{Vmatrix},\quad b^j_{\rangle} = \frac{16}{15}\frac{1}{x^2}\begin{Vmatrix} 2 \\ 5 \\ 1 \end{Vmatrix}. \qquad (32)$$

The matrix $\beta_n(x,\ y)$ is determined by Eq. (24) from [1]. The matrix $\gamma_k^{jn}(x)$ is presented in Appendix B.

Equation (23) is closed relative to the function $D(r,\ t)$. It should be solved with the initial conditions

$$D(r,\ 0) = D_0(r). \qquad (33)$$

The parameters ν, L, and ε are external parameters in this problem and should thus be assigned values.

Finding the general solution of the above equation is fairly complicated and falls outside the scope of the present work. It should be noted that asymptotic cases should be carefully investigated before posing this problem.

APPENDIX A

MATRICES a_{jk}^{lm}, b_{jk}^{lm}, c_{jk}^{lm}, a_{jk}^{m}, b_{jk}^{m}, c_{jk}^{m}

All matrix elements of the type a_{jk}^{lm} are polynomials in powers of n^2. For the sake of abbreviation, each nontrivial matrix element will be written in the form $|a_0,\ a_1,\ a_2,\ \ldots|$ or $|M(a_0,\ a_1,\ a_2,\ \ldots)|$, where M is a certain function of n^2 and $m \equiv n^2 - 1$; a_0, a_1, a_2, ... are integers which are coefficients of the polynomial $a_0 + a_1n^2 + a_2n^4 + \ldots$. For example, the entry $|1,\ 3,\ 5,\ 8|$ will denote a matrix element of the type $(1 + 3n^2 + 5n^4 + 8n^6)$; the entry $n^8(1,\ 0,\ 4)$ represents $(n^8 + 4n^{12})$; the entry $|0,\ 0,\ 1|$ represents n^4, etc.

We will write the four-dimensional matrices a_{jk}^{lm}, b_{jk}^{lm}, c_{jk}^{lm} in the form of one-dimensional matrices with fixed indices j, l, and m:

$$a_{ok}^{11} = \frac{m}{2}\|0|1,-3|0|0|0|0\|;\ a_{ok}^{12} = m\|-1,-1|-1,-1,6|0,0,4,-6|0|0|0\|;\ a_{ok}^{13} =$$

$$= 2m^2\|1|0,-2,-1|0,0,1,2|n^8(-1)|0|0\|; a_{ok}^{21} = \frac{1}{m}a_{ok}^{11};\ a_{ok}^{22} = m\|1|1,1|0,0,-3|0|0|0\|;$$

$a_{ok}^{23}=2m^2\|-1|0,2|0,0,-1|0|0|0\|$; $b_{ok}^{11}=-2a_{ok}^{11}$; $b_{ok}^{12}=-a_{ok}^{23}$; $b_{ok}^{21}=-\frac{2}{m}a_{ok}^{11}$; $b_{ok}^{22}=2\|-1,2|0,2,1|0|0|0|0\|$; $c_{ok}^{11}=mn^2\|-4|-3,5|0|0|0|0\|$; $c_{ok}^{12}=-n^2a_{ok}^{23}$; $c_{ok}^{21}=n^2\|-2|-3,3|0|0|0|0\|$; $c_{ok}^{22}=2n^2\|-1,-2|0,-2,1|0|0|0|0\|$; $a_{ok}^{l4}=b_{ok}^{l4}=c_{ok}^{l4}=b_{ok}^{l3}=c_{ok}^{l3}=b_{ok}^{l5}=a_{ok}^{l5}=c_{ok}^{l5}=0$; $a_{lk}^{11}=-\frac{3}{2}m^2\|0|-3,30,-35|0|0|0|0\|$; $a_{lk}^{12}=3m^3\|1,-5|-1,-25,70|0,-6,90,-140,|0|0|0\|$; $a_{lk}^{13}=6m^4\|1|0,-8,-15|0,-2,-5,70|0,0,-1,30,-70|0|0\|$; $a_{lk}^{14}=4n^2m^5\|0|6|0,-19,-15|0,-1,9,42|0,0,0,6,-28|0\|$; $a_{lk}^{15}=8n^4m^6\|0|0|1|0,-2,-1|0,0,1,2|n^8(-1)\|$; $a_{2k}^{11}=-\frac{3}{2}m^2\|0-1,5|0|0|0|0\|$; $a_{2k}^{12}=3m\|1,-2,1|0,0,20,-10|0,0,7,-32,15|0|0|0\|$; $a_{2k}^{13}=2n^2m\|6|0,-6,-18,6|0,8,-30,60,-20|0,1-12,36,-46,15|0|0\|$; $a_{2k}^{14}=4n^4m^5\|0|0|1|0,-2|0,0,1|0\|$; $a_{2k}^{15}=0$; $a_{3k}^{11}=\frac{3}{2}m^2\|0|3,-10,-35|0|0|0|0\|$; $a_{3k}^{12}=3m^3\|-3,-5|0,30,70|0,6,-30,-140|0|0|0\|$; $a_{3k}^{13}=6n^2m^4\|0|-6,-15|0,20,70|0,1,-10,-70|0|0\|$; $a_{3k}^{14}=4n^4m^5\|0|0|-3,-15|0,6,42|0,0,-2,-28|0\|$; $a_{3k}^{15}=8n^8m^6\|0|0|0|-1|0,2|0,0,-1\|$; $a_{1k}^{21}=\frac{9}{2}\frac{m^2}{n^2}\|0|-19,21,-10|0|0|0|0\|$; $a_{1k}^{22}=9\frac{m^3}{n^2}\|3,2|22,-31,-5|6,-59,72,-30|0|0|0\|$;

$a_{1k}^{23}=6\frac{m^3}{n^2}\|2,-3,-3|-19,-24,0,42|-6,0,96,-31,-45|0,0,36,-126,111,-30|0|0\|$; $a_{1k}^{24}=12\frac{m^3}{n^2}\|-1,-1,1|-6,6,10,0,-6|0,11,-12,1,-28,22|0,0,-6,5,-1,21,-15|n^8(6,-15,10,-2)|0\|$; $a_{1k}^{25}=24m^6\|0|1|0,-2,-1|0,0,1,2|n^8(-1)|0\|$; $a_{2k}^{21}=-\frac{3}{2}\frac{m^3}{n^2}\|0|1,3|0|0|0|0\|$; $a_{2k}^{22}=3\frac{m^3}{n^2}\|-3|-1,9|0,1,6|0|0|0\|$; $a_{2k}^{23}=3\frac{m^4}{n^2}\|0,-9|0,-1,8|n^6(1)|0|0\|$; $a_{2k}^{24}=12m^5\|0|1|0,-2|0,0,1|0|0\|$; $a_{2k}^{25}=0$; $a_{3k}^{21}=\frac{9}{2}\frac{m^2}{n^2}\|0|-1,-2,-15|0|0|0|0\|$; $a_{3k}^{22}=9\frac{m^3}{n^2}\|3,1|1,-8,5|0,-1,-4,-45|0|0|0\|$; $a_{3k}^{23}=6\frac{m^4}{n^2}\|-3,-3|0,27,36|0,3,-18,-25|0,0,0,-2,-45|0|0\|$; $a_{3k}^{24}=12m^5\|0|-3,-6|0,6,21|0,0,-2,-13|n^8(-3)|0\|$; $a_{3k}^{25}=-6ma_{2k}^{14}$; $b_{jk}^{l1}=-2a_{jk}^{l1}$; $b_{jk}^{l5}=0$; $b_{lk}^{12}=2m^2\|3,-18,15|0,-90,224,-114|n^2m^2(-18,141)|0|0|0\|$; $b_{lk}^{13}=4n^2m^2\|0,2|-6,48,-42,12|0,48,-124,114,-36|0,3,-34,84,-78,25|0|0\|$; $b_{lk}^{14}=2ma_{2k}^{14}$; $b_{2k}^{12}=2n^2m\|6,-3|0,-41,14|0,-18,34,-12|0|0|0\|$; $b_{2k}^{13}=4n^4m\|-1|-15,6,-1|-14,16,-9,2|m^4(-1)|0|0\|$; $b_{2k}^{14}=0$; $b_{3k}^{12}=2n^2\|18,18,-36,15|90,12,-360,383,-114|18,-63,-168,552,-486,141|0|0|0\|$; $b_{3k}^{13}=4n^4\|-3,-12,5|-45,-162,181,-76,12|-42,-120,360,-378,186,-36|-3,0,75,-200,225,-120,25|0|0\|$; $b_{3k}^{14}=8n^8\|1|28,-8,1|70,-56,28,-8,1|27,-50,55,-36,13,-2|m^6(1)|0\|$; $b_{1k}^{22}=\frac{6m^2}{n^2}\|9,-3,-6|85,-124,44,15|18,-99,162,-105,24|0|0|0\|$; $b_{1k}^{23}=12\frac{m^2}{n^2}\|1,0,-2,3|6,-13,1,34,-16|0,-11,31,-8,-22,12|mn^4(-6,15,-10,2)|0|0\|$; $b_{1k}^{24}=24m^6\|0|1|0,-2|0,0,1|0|0\|$; $b_{2k}^{22}=\frac{6m}{n^2}\|-1,1,3|0,-4,12,-5|0,-1,-3,3,-1|0|0|0\|$; $b_{2k}^{23}=12m\|0,0,-1|1,-4,-9,2|0,-2,$

$-6,4,-1|0|0|0\|;\ b_{2k}^{24}=0;\ b_{3k}^{22}=\frac{6}{n^2}\|-3,3,18,0,-3|0,-12,78,-88,32,1|0,-3,$

$0,-78,172,-133,36|0|0|0\|;\ b_{3k}^{23}=12\|0,0,-3,-10,3|3,-9,-45,-90,66,-15|$
$0,-6,-32,-20,60,-42,10|0,0,-1,15,-30,30,-15,3|0|0\|;\ b_{3k}^{24}=$
$24n^4\|0,0,1|0,0,28,-8,1||-1,6,55,-36,13,-2|0,2,15,-20,15,-6,1|0|0|;$

$c_{jk}^{l5}=0;\ c_{1k}^{l4}=n^2b_{1k}^{l4};\ c_{2k}^{l4}=0;\ c_{1k}^{11}=3m^2\|-4,32,-32|0,37,-130,93|0|0|0|$
$0\|;\ c_{1k}^{12}=2n^2m^2\|51,-94,33|48,-310,426,-174|0,-62,309,-432,185|0|0|0\|;$
$c_{1k}^{13}=4n^4m^2\|-12,4|-46,78,-54,14|-12,76,-156,132,-40|0,5,-42,96,-86,$
$27|0|0\|;\ c_{2k}^{11}=m\|6,-24,16|0,-3,62,-33|0|0|0|0\|;\ c_{2k}^{12}=2n^4m\|22,-5|40,-61,$
$18|8,-30,42,-14|0|0|0\|;\ c_{2k}^{13}=4n^4m\|6,-1|20,-15,6,-1|6,-14,16,-9,2|0,$
$-1,4,-6,4,-1|0|0\|;\ c_{3k}^{11}=3\|6,-20,-20,64,-32|0,-27,24,140,-228,93|0|0|$
$|0|0\|;\ c_{3k}^{12}=2n^2\|-72,-42,204,-156,33|-72,102,432,-972,699,-174|0,42,$
$-3,-448,912,-690,185|0|0|0\|;\ c_{3k}^{13}=4n^4\|18,69,-48,9|60,195,-358,265,-96,$
$14|18,30,-260,500,-462,214,-40|0,-3,-12,105,-240,255,-132,27|0|0\|;$
$c_{3k}^{14}=8n^8\|-8,1|-56,28,-8,1|-56,70,-56,28,-8,1|-8,27,-50,55,-36,13,$
$-2|n^2m^6(1)|0\|;\ c_{1k}^{21}=\frac{3m^2}{n^2}\|-56,44,0|-96,231,-201,66|0|0|0|0\|;\ c_{1k}^{22}=6\frac{m^2}{n^2}$
$\|-48,51,13,-26|-48,177,-200,28,33|0,30,-141,212,-129,28|0|0|0\|;\ c_{1k}^{23}=$
$=12m^2\|1,0,-14,5|6,-15,-31,52,-20|0,-11,23,4,-30,14|mn^4(-6,15,-10,$
$2)|0|0\|;\ c_{2k}^{21}=3\frac{m}{n^2}\|-4,6,-4|0,-1,8,-5|0|0|0|0\|;\ c_{2k}^{22}=6m\|7,-21,7|8,-18,20,$
$-7|n^2m^3(-1)|0|0|0\|;\ c_{2k}^{23}=12n^2m\|0,6,-1|1,16,-9,2|0,4,-6,4,-1|0|0|0\|;\ c_{3k}^{21}=$
$=\frac{3}{n^2}\|-12,30,-60,52,-16|0,-3,-30,200,-260,99|0|0|0|0\|;\ c_{3k}^{22}=6\|21,-87,$
$8,46,-21|24,-78,152,-88,-10,15|0,3,22,-138,232,-163,42|0|0|0\|;\ c_{3k}^{23}=12$
$n^2\|0,18,35,-30,5|3,51,125,-174,94,-19|0,12,0,-60,90,-54,12|n^4m^5(3)|0|0\|;$
$c_{3k}^{24}=24n^6\|0,-8,1|0,-56,28,-8,1|-1,-50,55,-36,13,-2|0,-6,15,-20,15,$
$-6,1|0|0\|.$

We will write the three-dimensional matrices a_{jk}^m, b_{jk}^m, c_{jk}^m in the form of one-dimensional matrices with fixed indices j and m:

$a_{1k}^1=\frac{3}{2}m^2\|2|1,-5|0|0\|;\ a_{1k}^2=m^2\|-1,-3|-1,-3,12|0,0,6,-10|0\|;\ a_{1k}^3=2m^3$
$\|1|0,-2,-1|0,0,1,2|n^8(-1)\|;\ a_{2k}^1=\frac{1}{2}m\|-2|-1,3|0|0\|;\ a_{2k}^2=m^2\|1|0,-2|0,0,1|$
$|0\|;\ a_{2k}^3=0;\ a_{3k}^1=\frac{3}{2}m\|2,2|1,-2,-5|0|0\|;\ a_{3k}^2=m^2\|-3,-3|0,6,12|0,0,-2,$
$-10|0\|;\ a_{3k}^3=2n^4m^3\|0|-1|0,2|0,0,-1\|;\ b_{1k}^1=4m^2\|0|1|0|0\|;\ b_{1k}^2=m^2\|2|6,-10|0,$
$-5,9|0\|;\ b_{1k}^3=2n^2m^3\|0|1|0,-2|0,0,1\|;\ b_{2k}^1=-\frac{1}{4m}b_{1k}^1;\ b_{2k}^2=m\|-1|-3,2|0,1,$
$-1|0\|;\ b_{2k}^3=0;\ b_{3k}^1=\|0|-3,0,4|0|0\|;\ b_{3k}^2=\|-3,0,2|-9,6,16,-10|3n^2m^2(1,3)$
$|0\|;\ b_{3k}^3=2n^4\|1|10,-5,1|5,-9,7,-2|n^2m^3(1)\|;\ c_{1k}^1=0;\ c_{1k}^2=m^2\|-6|-3,7|0|0\|;$
$c_{1k}^3=2m^3\|1|0,-2|0,0,1|0\|;\ c_{2k}^1=0;\ c_{2k}^2=m\|2|1,-1|0|0\|;\ c_{2k}^3=0;\ c_{3k}^1=0;$
$c_{3k}^2=\|6,0,-6|3,-3,-9,7|0|0\|;\ c_{3k}^3=2n^4\|-5,1|-9,7,-2|m^3(1)|0\|.$

APPENDIX B
MATRIX $\gamma_k^{jn}(x)$

The matrix $\gamma_k^{jn}(x)$, $k = -6, -5, \ldots, 7$; $j = 1, 2, 3$; $n = 1, 2, 3, 4$, is conveniently represented by superimposing two matrices $\bar{\gamma}_p^{jn}(x)$, $p = -3, -2, \ldots, 2, 3$; and $\bar{\bar{\gamma}}_q^{jn}(x)$, $q = -4, -3, \ldots, 1, 2$, according to the following rule:

$$\gamma_k^{jn}(x) = \begin{cases} x^{-3}\bar{\gamma}_{k/2}^{jn}(x), & \text{if} \quad k \quad \text{is even,} \\ x^{-6}\bar{\bar{\gamma}}_{k/2-3}^{jn}(x), & \text{if} \quad k \quad \text{is odd.} \end{cases} \tag{B.1}$$

The elements of the matrices $\bar{\gamma}$ and $\bar{\bar{\gamma}}$ are expressions of the type $\Lambda^m(b_0 + b_1x^2 + b_2x^4 + \ldots)$, where $\Lambda = 1 - x^2$; m is a certain integer; $b_0, b_1, b_2, \ldots$ are numerical coefficients. For the sake of abbreviation, the polynomial in x^2 will be represented in accordance with the entry $(b_0, b_1, b_2, \ldots)$.

We will write the three-dimensional matrices $\bar{\gamma}$ and $\bar{\bar{\gamma}}$ in the form of two-dimensional matrices with a fixed index n:

$$\bar{\gamma}_p^{j1} = \left|\begin{array}{lll} \frac{1}{2}\Lambda^4(-1,-1) & \Lambda^4(4) & \Lambda^4(1,1) \\ \frac{1}{2}\Lambda^2(-7,6,1) & \Lambda^2(-20,-4) & \Lambda^2(-5,-2,-1) \\ 15,-3,1,3 & 36,-8,4 & \Lambda(6,4,6) \\ -19,-14,-7 & -28,-12 & 2,8,14 \\ \frac{1}{2}(19,11) & 8 & -7,-11 \\ \frac{1}{2}(-3) & 0 & 3 \\ 0 & 0 & 0 \end{array}\right|;$$

$$\bar{\gamma}_p^{j2} = \left|\begin{array}{lll} \frac{1}{2}\Lambda^4(-1,-1) & \Lambda^4(4) & \Lambda^4(1,1) \\ \frac{1}{2}\Lambda^2(29,-34,5) & \Lambda^2(28,108) & \Lambda^2(7,22,-5) \\ -45,9,-3,-9 & -108,14,-12 & \Lambda(-18,-12,-18) \\ 53,50,17 & 116,-60 & 2,-40,-34 \\ \frac{1}{2}(-53,-29) & -40 & 17,29 \\ \frac{1}{2}(9) & 0 & -9 \\ 0 & 0 & 0 \end{array}\right|;$$

$$\bar{\gamma}_p^{j3}=\left|\begin{array}{lll}
\frac{1}{2}\,\Lambda^4(-1,-1) & \Lambda^4(4) & \Lambda^4(1,1) \\
\frac{1}{2}\,\Lambda^2(-7,22,17) & \Lambda^2(-20,-68) & \Lambda^2(-5,-18,-17) \\
15,-27,-15,-21 & 36,-72,-60 & \Lambda(6,-12,-42) \\
-19,10,17 & -28,116 & 2,24,-34 \\
\frac{1}{2}\,(19,-5) & 8 & -7{,}5 \\
\frac{1}{2}\,(-3) & 0 & 3 \\
0 & 0 & 0
\end{array}\right|;$$

$$\bar{\gamma}_p^{j4}=\left|\begin{array}{lll}
\frac{1}{2}\,\Lambda^4(-1,-1) & \Lambda^4(4) & \Lambda^4(1,1) \\
\frac{1}{2}\,\Lambda^2(29,54,-35) & \Lambda^2(28,-52) & \Lambda^2(7,-2,35) \\
-45,69,37,51 & -108,184,148 & \Lambda(-18,28,102) \\
53,-74,-43 & 116,-60 & 2,-16,86 \\
\frac{1}{2}\,(-53,11) & -40 & 17,-11 \\
\frac{1}{2}\,(9) & 0 & -9 \\
0 & 0 & 0
\end{array}\right|;$$

$$\bar{\bar{\gamma}}_q^{j1}=\left|\begin{array}{lll}
0 & 0 & 0 \\
\frac{1}{2}\,\Lambda^4(-1,-3) & \Lambda^3(4,8) & \Lambda^3(1,2,1) \\
\frac{1}{2}\,\Lambda^3(-5,-3) & \Lambda(-16,-28,-4) & \Lambda(-7,-7,-1,-1) \\
15,3,5,9 & 24,36,20 & 18,14,10,6 \\
-25,-26,-21 & -16,-20 & -22,-24,-14 \\
\frac{1}{2}\,(35,33) & 4 & 13,11 \\
\frac{1}{2}\,(-9) & 0 & -3
\end{array}\right|;$$

$$\bar{\bar{\gamma}}_q^{j2}=\left|\begin{array}{lll}
0 & 0 & 0 \\
\frac{1}{2}\,\Lambda^4(-1,-3) & \Lambda^3(4,8) & \Lambda^3(1,2,1) \\
\frac{1}{2}\,\Lambda(31,-45,-35,-15) & \Lambda(-16,132,92) & \Lambda(5,29,19,-5) \\
-57,31,-11,-27 & 24,-284,-44 & -30,-74,-22,-18 \\
83,50,51 & -16,140 & 50,80,34 \\
\frac{1}{2}\,(-109,-87) & 4 & -35,-29 \\
\frac{1}{2}\,(27) & 0 & 9
\end{array}\right|;$$

$$\overline{\overline{\gamma}}_q^{j3} = \left|\begin{array}{lll} 0 & 0 & 0 \\ \frac{1}{2}\Lambda^4(-1,-3) & \Lambda^3(4,8) & \Lambda^3(1,2,1) \\ \frac{1}{2}\Lambda(-5,23,33,-51) & \Lambda(-16,-92,-132) & \Lambda(-7,-23,-33,-17) \\ 15,-37,-11,-63 & 24,164,-44 & 18,30,-22,-42 \\ -25,30,51 & -16,-84 & -22,-8,34 \\ \frac{1}{2}(35,-15) & 4 & 13,-5 \\ \frac{1}{2}(-9) & 0 & -3 \end{array}\right|;$$

$$\overline{\overline{\gamma}}_q^{j4} = \left|\begin{array}{lll} 0 & 0 & 0 \\ \frac{1}{2}\Lambda^4(-1,-3) & \Lambda^3(4,8) & \Lambda^3(1,2,1) \\ \frac{1}{2}\Lambda(31,43,13,105) & \Lambda(-16,-28,92) & \Lambda(5,5,35,35) \\ -57,3,29,153 & 24,36,116 & -30,14,58,102 \\ 83,-26,-129 & -16,-20 & 50,-24,-86 \\ \frac{1}{2}(-109,33) & 4 & -35,11 \\ \frac{1}{2}(27) & 0 & 9 \end{array}\right|.$$

NOTATION

$\Delta V(\vec{r})$, difference in velocity at the points $\vec{r}$, 0; $\vec{\theta}$, η, arguments of the characteristic function φ; $[\ \ ]_0 = [\ \ ]_{\vec{\theta}=0,\,\eta=0}$;

$$\vec{v} = \frac{\vec{r}}{r};\ \vec{\lambda} = \frac{\vec{y}}{y};\ \vec{\varkappa} = \frac{\vec{x}}{x};\ \vec{\mu} = \frac{\vec{z}}{z};\ \rho = \frac{y}{r};\ \sigma = \frac{x}{r};\ \xi = \frac{z}{r};$$

$$s = (\vec{v}\cdot\vec{\lambda});\ t = (\vec{v}\cdot\vec{\varkappa});\ l = (\vec{v}\cdot\vec{\mu});\ \delta = (\vec{\varkappa}\cdot\vec{\lambda});\ \gamma = (\vec{\mu}\cdot\vec{\lambda});\ \omega = (\vec{\varkappa}\cdot\vec{\mu});$$

$$\Delta_{\alpha\beta} = \lambda_\alpha\lambda_\beta - \delta_{\alpha\beta};\ \Delta_\alpha = s\lambda_\alpha - v_\alpha;\ \delta_\alpha = v_\alpha - \rho\lambda_\alpha;$$

$$n = \left[1 + \frac{1}{y\,[\ln D(y)]'_y}\right]^{1/2};\ m = n^2 - 1;\ \gamma(y) = -y\,[\ln D''_y(y)]'_y;$$

$$\beta(y) = -y\,[\ln D'_y(y)]'_y;\quad \varepsilon_d = -\frac{15}{2}\nu D''_r(0);$$

ν, kinematic viscosity coefficient; L, external scale of turbulence; ε, specific rate of turbulent energy pumping.

LITERATURE CITED

1. V. A. Sosinovich, "Closed equation for probability distribution of velocity and temperature differences at two points in an isotropic turbulent flow of an incompressible fluid," Inzh.-Fiz. Zh., 36, No. 6, 972-979 (1979).
2. A. S. Monin and A. M. Yaglom, Statistical Fluid Mechanics, Vol. 2, MIT Press (1975).
3. F. N. Frenkiel and P. S. Klebanoff, "Higher-order correlations in a turbulent field," Phys. Fluids, 10, No. 3, 507-520 (1963).
4. M. S. Uberoi, "Energy transfer in isotropic turbulence," Phys. Fluids, 6, No. 8, 1048-1056 (1963).
5. C. W. Van Atta and T. T. Yeh, "Correlation measurements in turbulence using digital Fourier analysis," Phys. Fluids Suppl., 12, No. 11, 264-269 (1969).

6. T. Tatsumi, Sh. Kida, and J. Mizushima, "The multiple-scale cumulant expansion for isotropic turbulence," J. Fluid Mech., 85, No. 1, 97-142 (1978).
7. E. A. Novikov, "Functionals and the random forces method in the theory of turbulence," Zh. Eksp. Teor. Fiz., 47, No. 5 (11), 1919-1926 (1964).
8. A. N. Kolmogorov, "Energy dissipation in locally isotropic turbulence," Dokl. Akad. Nauk SSSR, 32, No. 1, 19-21 (1941).
9. P. C. Leslie, Developments in the Theory of Turbulence, Clarendon Press, Oxford (1973).

DEPTH OF PENETRATION OF SOLID PARTICLES INJECTED INTO A GAS FLOW

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The depth of penetration of a spherical particle into a uniform transverse flow was determined experimentally and theoretically for both a continuous medium and a free molecule flow.

In solving a number of practical problems of two-phase hydrodynamics, it is often necessary to determine the depth of penetration of particles introduced into the flow.

Let us examine this problem for a continuous medium. The following classical expression for depth of penetration S is known for small Reynolds numbers $\mathrm{Re} \ll 1$:

$$S = \frac{\rho_p d^2 V_0}{18\mu}. \quad (1)$$

However, S is independent of the velocity of the transverse flow and, as was shown in [1], when Re > 1 substantial corrections of (1) are necessary to determine the penetration depth.

Let us choose a coordinate system such that the x axis will be perpendicular to the flow and the y axis will be parallel to the flow. Then the equations of motion of the particles will have the following form:

$$\frac{dx}{dt} = V_p, \quad (2)$$

$$\frac{dV_p}{dt} = -\frac{3C_D \,\mathrm{Re}\,\mu}{4\rho_p d^2} V_p, \quad (3)$$

$$\frac{dU_p}{dt} = \frac{3C_D \,\mathrm{Re}\,\mu}{4\rho_p d^2}(U_q - U_p), \quad (4)$$

with the initial conditions $t = 0$, $V_p = V_0$, $x = 0$, and $U_p = 0$. The quantity C_D is a function of the Reynolds number

$$\mathrm{Re} = \frac{\rho_q d\,[(U_q - U_p)^2 + V_p^2]^{1/2}}{\mu}.$$

We will evaluate the braking distance on the basis of (2) and (3). The braking time τ and distance S are equal to the following, respectively, in order of magnitude

$$\tau \sim \frac{V_p}{\left.\frac{dV}{dt}\right|_{t=0}} = \frac{\rho_p d^2}{C_D(\mathrm{Re}_0)\,\mathrm{Re}_0\mu},$$

$$S \sim V_0\tau = \frac{\rho_p d^2 V_0}{C_D(\mathrm{Re}_0)\,\mathrm{Re}_0\mu}. \quad (5)$$

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